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# Non-static plane symmetric dark energy universe with cosmic strings in general relativity

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**Abstract**

In this paper, we have studied the non-static plane symmetric universe filled with dark energy and one-dimensional cosmic string in general relativity. We have solved the field equations by taking the hybrid scale factor. We have discussed some physical and geometrical properties of the obtained model such as spatial volume (*V*), deceleration parameter (*q*), expansion scalar ($\theta$), Hubble parameter (*H*), shear scalar ($\sigma$),anisotropic parameter ($A_h$), state finder parameters (*r*, *s*) and squared speed of sound($v_s^2$). Also, it is observed that for our model the obtained values coincide with the present observations.

**Key words**: Non-static plane symmetric, dark energy, string tension density, general relativity theory of gravitation.

## Introduction

One of the outstanding developments in cosmology is the discovery of the accelerated expansion of the universe which is believed to be driven by some exotic dark energy (Reiss *et al.,* 1998; Perlmutter *et al.,* 1999; Spergel 2003; Copeland *et al.,* 2006; Spergel *et al.,* 2007). The thermo-dynamical studies of dark energy reveal that the constituents of dark energy may be massless particles (bosons or fermions) whose collective behavior resembles with a kind of radiation fluid having negative pressure. Also, it is commonly believed by the cosmological community that this unknown exotic physical entity known as dark energy is a kind of repulsive force which acts as antigravity responsible for gearing up the universe.

The Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment suggests that 73% content of the universe is in the form of dark energy, 23% is in the form of non-baryonic dark matter and rest 4% is in the form of usual baryonic (normal) matter as well as radiation. Different dark energy models are distinguished by focusing on the parameter value of the equation of state (EoS) $w_{de} = \frac{p_{de}}{r_{de}}$ ($p_{de}$ =pressure of dark energy and ($\rho_{de}$ = density of dark energy) which is not necessarily a constant. The cosmological constant (Λ) is one of the candidate of dark energy which is mathematically equal to vacuum energy ($w_{de}$ =-1) (Sahni *et al., 2008*; Vinutha *et al., 2018*). In general, EoS parameter has to be considered as a constant in the absence of observational evidence and has values -1, 0, $\frac{1}{3}$for vacuum, dust and radiation respectively (Kujat *et al., 2002*; Bartelmann *et al., 2005*). EoS parameter

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$w_{de}$ is a function of cosmic time or redshift (Jimenez *et al., 2003*; Das *et al., 2005*). To understand the nature of dark energy phenomenon, various dynamic dark energy models has been proposed, which can be described by the equation of state parameter.

In recent years, the study of cosmic strings is one of the great interests. Cosmic strings attracted considerable attention because they believed that they were working in the construction of the early stages of the universe. Cosmic strings can be created during phase changes in the early era (Kibble 1976) and act as the source of the gravitational field (Letelier 1980). The density perturbations to form large-scale structures of the universe is believed to be one of the strings in the corners of the properties. The formation of cosmic strings is somewhat analogous to the imperfections that form between crystal grains in solidifying liquids, or the cracks that form when water freezes into ice. The phase transitions leading to the production of cosmic strings are likely to have occurred during earliest moments of the universe evolution, just after cosmological inflation. Letelier 1983, Maharaj and Beesham 1988, Krori *et al., 1990*, 1994, Raj and Shuchi 2001, Bhattacharjee and Baruah 2001, Mahanta and Mukharjee 2001 and Reddy 2003 are some of the authors who have studied various aspects of string cosmological models in general relativity. Anisotropic non-static plane symmetric cosmological models of the universe have some interesting applications in cosmology. Plane symmetric space -times with various matter distributions have been discussed in GR owing to possible applications to astrophysics and cosmology (Maharaj and Beesham 1988; Krori *et al., 1990*, 1994; Raj and Shuchi 2001). Several researchers in literature have investigated anisotropic DE models within the frameworks of general relativity (Krori *et al., 1990*, 1994; Reddy 2003; Raychaudhuri 2016; Vinutha *et al., 2018*; Thomas *et al., 2019*). In the present work, we have investigated non-static plane symmetric dark energy string cosmological model in G.R. In addition to the anisotropic DE fluid, cosmic strings assigned along *z*-direction are also considered to comprise some anisotropic effect (Mishra *et al., 2017*).

The paper is organized as follows. In section 2, we discuss about field equations and their solutions. In section 3 some important properties of the models like equation of state parameter($w_{de}$), square speed of sound $v_s^2$and state finder parameter $\{r, s\}$ are discussed, we summarize the results and graphical representation for the model in the last section.

**Field Equations and Solutions:**

**Metric and Basic Field Equations:** The Einstein's field equations are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -8pGT_{ij}, \quad (1)$$

where$R_{ij}$, $g_{ij}$and $R$ are the Ricci tensor, metric tensor and the Ricci scalar respectively. Here we consider 8πG = 1. The energy momentum tensor $T_{ij}$ for a given environment of two non-interacting fluids is given by

$$T_{ij} = T_{ij}^{de} + T_{ij}^{l}, \quad (2)$$

where $T_{ij}^{de}$ is the stress energy tensor of the dark energy and $T_{ij}^{l}$ is the stress energy tensor of one dimensional cosmic string. Also, we have energy conservation equation as

$$T^{ij}_{;j} = 0, \quad (3)$$

here semicolon denote covariant differentiation. The energy-momentum tensor of the dark energy is given by

$$T_{ij}^{de} = (\rho_{de} + p_{de})u_i u_j - p_{de} g_{ij} \quad (4)$$

where $p_{de}$ and $\rho_{de}$ are pressure and density of the dark energy respectively.

For fluid containing one dimensional cosmic string, the energy momentum tensor is given by (Stachel 1980; Mishra *et al., 2017*)

$$T_{ij}^{\lambda} = (\rho + p)u_i u_j - p_{de} g_{ij} - \lambda x_i x_j \quad (5)$$

here $u_i u_j = 1, x_i x_j = -1$ (along *z*-direction). In the co-moving coordinate system $u^i$ is the four-velocity vector and *p* is the isotropic pressure of the fluid. $\rho$ is the proper density and is composed of energy density due to massive particle and string tension density. In the absence of any string phase, the total contribution to the baryonic energy density comes from particles only. In contrast to isotropic pressure of usual cosmic fluid, we wish to incorporate some degree of anisotropy in the dark energy pressure.

Now we consider the non-static plane symmetric metric of the form

$$ds^2 = e^{2A}[dt^2 - dr^2 - r^2 dq^2 - B^2 dz^2], \quad (6)$$

where *A* and *B* are functions of cosmic time '*t*'only.

By using equations (1), (2), (4) and (5), we get field equations as follows

$$e^{-2A}\left[2\ddot{A} + \frac{\ddot{B}}{B} + \dot{A}^2 + 2\frac{\dot{A}\dot{B}}{B}\right] = -p_{de} - p \quad (7)$$

$$e^{-2A}\left[2\ddot{A} + \dot{A}^2\right] = -p_{de} - p + \lambda, \quad (8)$$

$$e^{-2A}\left[3\dot{A}^2 + 2\frac{\dot{A}\dot{B}}{B}\right] = \rho_{de} + \rho, \quad (9)$$

The energy conservation equation (3), yields

$$\dot{\rho_{de}} + \dot{\rho} + (\rho_{de} + \rho)(H_2 + 4H_1) + 2(p_{de} + p)H_1 + (p_{de} + p)(H_1 + H_2) - (\rho_{de} + \rho)H_1 - \lambda(H_1 + H_2) = 0, \quad (10)$$

where $H_1$ and $H_2$ are directional Hubble's parameters and dot represents derivative with respect to cosmic time '*t*'.

**Solutions of the field equations:** From the field equations (7) to (9) we have three independent equations with seven unknowns *A*, *B*, $\rho$, $\rho_{de}$, $p_{de}$, *p* and $\lambda$. In order to find a deterministic solution, we take the following two physically valid conditions.

(i) We consider the cosmological scale factor as a hybrid expansion law (Saha *et al., 2012*)

$$R(t) = me^{at}t^b, \quad (11)$$

where $a \geq 0$, $b \geq 0$ and $m > 0$ are constants.

(ii) We take the shear scalar $\sigma$ in the model to be proportional to the expansion scalar $\theta$, this condition leads to (Collins *et al., 1980*)

$$e^A = B^l, \quad (12)$$

where *l*=1 is an arbitrary constant, which preserves the anisotropic nature of the model.

Using equation (12) we get,

$$\dot{A} = l\frac{\dot{B}}{B}. \quad (13)$$

From equation (13)

$$H_1 = lH_2 \quad (14)$$

The mean Hubble parameter is,

$$H = \frac{1}{3}(H_1 + H_2 + H_3). \quad (15)$$

Where $H_1 = \frac{\dot{A}}{A}$, and $H_2 = H_3 = \frac{\dot{B}}{B}$ are directional Hubble parameters in the direction of *r*, $\theta$ and *z* respectively.

The Hubble's parameter *H* for the model is obtained as,

$$H = \frac{\dot{R}}{R} = a + \frac{b}{t}. \quad (16)$$

Then from equations (14) and (15) we get,

$$H_1 = \frac{3l}{l+2}H, H_2 = H_3 = \frac{3}{l+2}H, \quad (17)$$

and from equations (16) and (17), the metric potentials are obtained as

$$A = \log(me^{at}t^b)^{\frac{3l}{l+2}} \quad (18)$$

$$B = (me^{at}t^b)^{\frac{3}{l+2}} \quad (19)$$

the metric equation (6) with the help of equations (18) and (19) can be written as

$$ds^2 = e^{\log(me^{at}t^b)^{\frac{6l}{l+2}}}\left[dt^2 - dr^2 - r^2 d\theta^2 - (me^{at}t^b)^{\frac{6}{l+2}} dz^2\right]. \quad (20)$$

From equations (7) and (8) we get the string tension density as $\lambda$

$$\lambda = RS$$

where $R = \frac{-18\,(me^{at}t^{b})^{\frac{-6l}{l+2}}}{(l+2)^2 t^2}$,

and

$$S = \left(l+\tfrac{1}{2}\right)b^2 + \left(2alt + at - \tfrac{1}{6}l - \tfrac{1}{3}\right)b + a^2t^2\left(l+\tfrac{1}{2}\right)$$

We consider $= \alpha\rho \Rightarrow \rho = \frac{\lambda}{\alpha}$ (22)

where $\alpha$ is non evolving state parameter.

From equations (21) and (22) the proper density $\rho$ is given by,

$$\rho = R_1 S_1.$$

Where $R_1 = \frac{-18\,(me^{at}t^{b})^{\frac{-6l}{l+2}}}{(l+2)^2 t^2 \alpha}$,

and

$$S_1 = \left(l+\tfrac{1}{2}\right)b^2 + \left(2alt + at - \tfrac{1}{6}l - \tfrac{1}{3}\right)b + a^2t^2\left(l+\tfrac{1}{2}\right)$$

We consider, $p = \omega\rho$, (24)

where $\omega$ is non evolving state parameter.

From equations (23) and (24), we obtain the isotropic pressure of the fluid $p$ as

$p = R_2 S_2$. (25)

Where $R_2 = \frac{-18\,\omega(me^{at}t^{b})^{\frac{-6l}{l+2}}}{(l+2)^2 t^2 \alpha}$,

and

$$S_2 = \left(l+\tfrac{1}{2}\right)b^2 + \left(2alt + at - \tfrac{1}{6}l - \tfrac{1}{3}\right)b + a^2t^2\left(l+\tfrac{1}{2}\right)$$

From equations (9) and (23), we obtain the density of dark energy $\rho_{de}$ as

$$\rho_{de} = \frac{27\,(me^{at}t^{b})^{\frac{-6l}{l+2}}}{(l+2)^2 t^2 \alpha}\left[\alpha(at+b)^2 l^2 + \left(\left(\frac{2\alpha}{3} + \frac{2}{3}b^2 + \left(-\frac{1}{9} + \frac{4\alpha(\alpha+1)l}{3}\right)b + \frac{2\alpha^2t^2(\alpha+1)}{3}\right)l + \frac{b^2}{3} + \left(\frac{2at}{3} - \frac{2}{9}\right)b + \frac{a^2t^2}{3}\right)\right]$$

(26)

From equations (8), (21) and (25), we obtain the pressure of dark energy $p_{de}$ as

$$p_{de} = \frac{9\,(me^{at}t^{b})^{\frac{-6l}{l+2}}}{(l+2)^2 t^2 \alpha}\left[((l+1)^2\alpha - 2\omega l - \omega)b^2\left(2at((l+2)^2\alpha - 2\omega l - \omega) - \frac{2(l+2)\left(\left(l+\frac{1}{2}\right)\alpha - \frac{\omega}{2}\right)}{3}\right)b[+a^2t^2((l+2)^2\alpha - 2\omega l - \omega)]\right].$$

(27)

From equations (26) and (27), we obtain the equation of state parameter (EoS) $\omega_{de}$ as

$$\omega_{de} = \frac{S_3}{R_3} \qquad (28)$$

Where

$$S_3 = (-3\alpha l^2 + 6l(\omega - \alpha) - 3\alpha + 3\omega)b^2 - 3a^2t^2(\alpha l^2 + 2l(\alpha - \omega) + \alpha - \omega) + \left((2 - 6al)\alpha l^2 + (-12\,\alpha t(\alpha - \omega) + 5\alpha - \omega)l - 6(\alpha - \omega)\left(at - \tfrac{1}{3}\right)\right)b$$

and (21)

$$R_3 = (3 + 9\alpha l^2 + (6\alpha + 6)l)b^2 + 9a^2t^2\left(\frac{1}{3} + \alpha l^2 + \left(\frac{2\alpha}{3} + \frac{2}{3}\right)l\right) + (18a\alpha l^2 t + (-1 + 12at(\alpha + 1))l + 6at - 2)b$$

**Some other important properties of the model:** The spatial volume of the model with equation (20) is given by,

$V = R^3 = (me^{at}t^{b})^3$. (29)

The expansion scalar q for the model is obtained as,

$\theta = u^i_{;i} = 3H = 3\left(a + \frac{b}{t}\right)$, (30)

(23)

we observe that when $t \to 0$, $\theta \to 0$ and this indicates the inflationary scenario at early stages of the universe.

The shear scalar $\sigma$ for the model is obtained as,

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{3}\sum_{i=1}^{3} H_i^2 - \frac{1}{6}\theta^2 = 3\left(\frac{l^2-2l+1}{(l+2)^2}\right)\left(a + \frac{b}{t}\right)^2.$$

(31)

The average anisotropic parameter $A_h$ for our model is given by

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\frac{(H_i - H)^2}{H^2} = 2\frac{(l-1)^2}{(l+2)^2}. \qquad (32)$$

For the obtained model, throughout the discussions of graphical representations of the physical parameters, we fix the constants as: $a$ = 0.055, $b$ = 0.29, $l$ = -12.5, a = 0.110, $m$=1, w = 1 and the cosmic time $t$ in billion years.

The deceleration parameter $q$ acts as the indicator of the existence of inflation of the model. If $q > 0$ the model decelerates in the standard way while $q < 0$ indicates inflation of the universe (Riess *et al., 1998*; Bennett *et al., 2003*). Recent observations of SNe Ia, demonstrated that the present universe is accelerating and value of deceleration parameter lies in the range of $-1 \le q < 0$ (Ade *et al., 2014*) and it is defined as follows.

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -1 + \frac{b}{(at+b)^2}. \qquad (33)$$

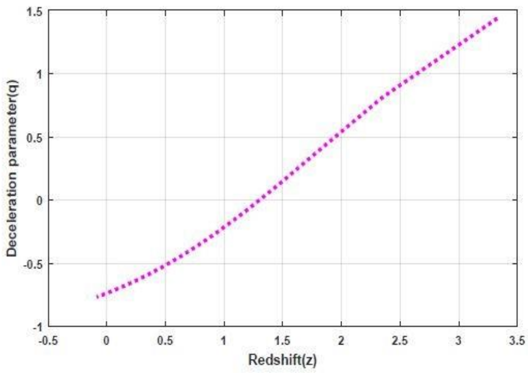


**Figure 1**: Plot of $q$ versusredshift($z$)

In figure 1 the present deceleration parameter is plotted with respect to redshift and it is observed that$q_0 \approx -0.73$, which match the observed value (Cunha *et al., 2009*) and also noticed that

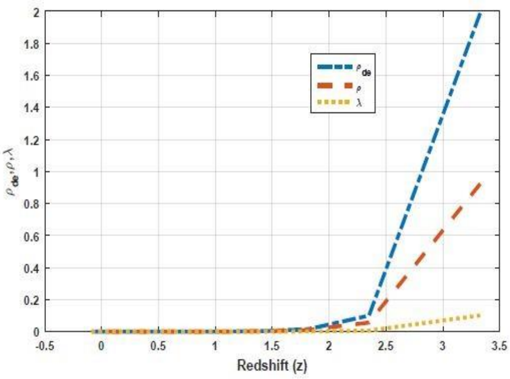


**Figure 2**: Plot of $\rho_{de}$, $\lambda$ and $\rho$ versus red shift ($z$).

(i) $q < 0$ for $z < 1.25$ indicates that the universe is expanding in accelerating rate at present epoch and late time.

(ii) $q > 0$ for $z > 1.25$ indicates that the model was decelerating at early stage of the universe.

(iii) For $q = 0$ at $z \approx 1.25$, shows the transition from early deceleration to late time inflation of the universe.

From figure 2, it is clear that dark energy density is an increasing function of the red shift ($z$) and it is positive. Further, the dark energy density ($\rho_{de}$) dominates both density of dark matter (r) and string tension density ($\lambda$).

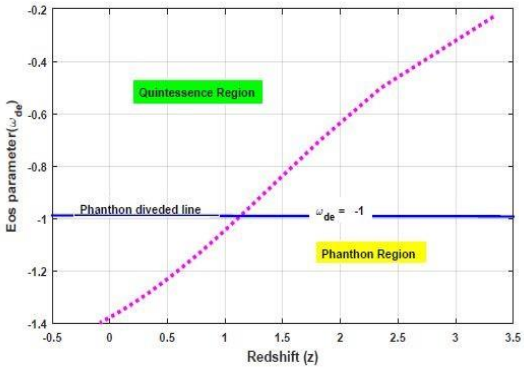


Figure 3: Plot of $w_{de}$ versus redshift($z$.)

**Equation of state parameter:**

From equation (28), It can be seen that the equation of state parameter ($\omega_{de}$) is a function of cosmic time $t$ and in figure 3, we plotted $\omega_{de}$ against redshift($z$) and it crosses the phantom divide line ($\omega_{de} = -1$). So $\omega_{de}$ is varying from quintessence to phantom and it has quintom like behavior.

**Squared speed of sound ($v_s^2$):** To analyze the stability of DE models the squared speed of sound ($v_s^2$) parameter is the best tool. A positive value of this parameter indicates stable behavior while negative value indicates instability. The squared speed of the sound is defined as follows (Myung 2007),

$$v_s^2 = \frac{\dot{p}_{de}}{\dot{\rho}_{de}} = \frac{R_4}{S_4} \qquad (34)$$

where $\dot{p}_{de}$ and $\dot{\rho}_{de}$ are cosmic time derivatives of pressure and density of dark energy, respectively and

$$v_s^2 = \frac{1}{S_4}\Big[-9lb^3(\alpha l^2 + 2l(\alpha - \omega) + \alpha - \omega) + \Big((-27a\alpha t + 3\alpha)l^3 + (-54at(\alpha - \omega) + 3\alpha + 3\omega)l^2 - 27(\alpha - \omega)\left(at + \frac{1}{3}\right)l - 6(\alpha - \omega)\Big)b^2 + \Big((-27a^2t^2\alpha + 3at\alpha + 2\alpha)l^3 + (-54t^2\alpha^2(\alpha - \omega) + 3ta(\alpha + \omega) + 9\alpha - \omega)l^2 + (-27a^2t^2(\alpha - \omega) - 9al(\alpha - \omega) + 12\alpha - 4\omega)l - 6\left(at - \frac{2}{3}\right)(\alpha - \omega)\Big)b - 9lt^3a^3(\alpha l^2 + 2(\alpha - \omega)l + \alpha - \omega)\Big] \qquad (35)$$

Where

$$S_4 = (27\alpha l^3 + 18\, l^2(\alpha + 1) + 9l)b^3 + (6 + (81at\alpha + 9\alpha)l^3 + (54at(\alpha + 1) + 24\alpha + 3)l^2 + (27at + 12\alpha + 9)l)b^2 + \Big((81a^2\alpha t^2 + 9a\alpha t)l^3 + (-1 + 54t^2(\alpha + 1)\alpha^2 + (24\alpha + 3)t\alpha)l^2 + (-4 + 27a^2t^2 + (12\alpha + 9)t\alpha)l + 6at - 4\Big)b + 27la^3t^3\left(\frac{1}{3} + l^2 + \left(\frac{2\alpha}{3} + \frac{2}{3}\right)l\right)$$

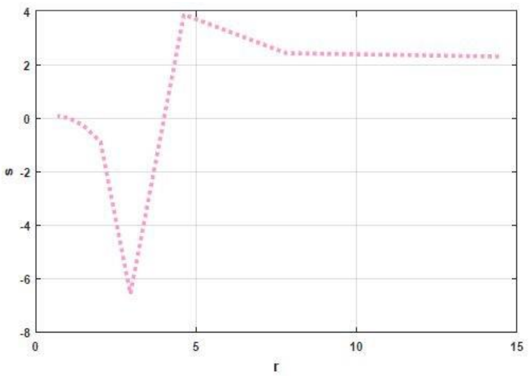


**Figure 4**: Plot of $v^2$ versus red shift($z$).

The behavior of $v_s^2$ with the equations (26), (27) and (34) is displayed against redshift($z$) is shown in figure 4 and we observe that $v_s^2$ is positive for the model which reveals that the model is stable at early time.

**State finder parameters:** Several DE models have been introduced in order to explain the current rapid expansion of the universe. To differentiate several candidates of DE models, introduced new geometrical parameters, named state finder parameters pair Error!, where $r$ is generated from the third derivative with respect to the cosmic time $t$ of the scale factor $R$ and s is a simple combination of $r$ and the deceleration parameter $q$. State finder parameters are defined as follows,

$$r=\frac{\dddot{R}}{RH^3},\quad s=\frac{r-1}{3(q-\frac{1}{2})}. \quad (36)$$

Using equations (11) and (16) in (36) we get,

$$r=1-\frac{b(3at+3b-2)}{(at+b)^3}. \quad (37)$$

From equations (33), (36) and (37) we get,

$$s=\frac{2b(2-3at-3b)}{(6b-9(at+b)^2)(at+b)}.$$

In the $r$-$s$ plane$\{r,s\}$ =(1,0), (1,1) indicates ΛCDM model and CDM model respectively (Huang *et al.,* 2008). While $s$>0 and $r$<1 represents the region of phantom and quint essences dark energy model eras. From figure 5 and we have observed that the values of state finder pair becomes $r$ = 1, $s$ = 0 at late time and is consistent with standard ΛCDM model.

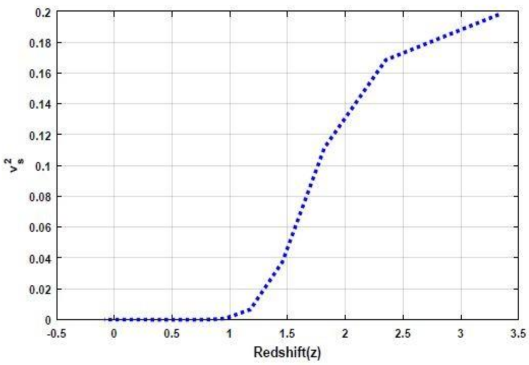


**Figure 5**: Plot of $r$ versus $s$.

$w_{de}$ - $w_{de}'$ **Plane analysis:** The $w_{de}$ - $w_{de}'$ plane analysis is firstly introduced by Caldwell and Linder 2005 which is a very useful tool for testing various behaviors of quintessence scalar field dark energy models through this plane. Basically, it has been used to distinguish different DE models through trajectories on its plane. Initially, this area belongs to the region ($w_{de}$<0,$w_{de}'$>0) corresponds to thawing region while area under the region ($w_{de}'<0, w_{de}<0$) corresponds to the freezing region. Differentiating EoS parameter $w_{de}=\frac{p_{de}}{r_{de}}$ with respect to ($lnR$) we get, $\omega_{de}'=\frac{\dot{p}_{de}\rho_{de}-p_{de}\dot{\rho}_{de}}{H\rho_{de}^2}=\frac{R_5}{S_5}$

Where

$R_5=2tabl\alpha(l+2)(3l+2)(2l\alpha+\alpha-\omega+1)$

And

$$S_5=27\left(\left(\frac{1}{3}+\alpha t^2+\left(\frac{2\alpha}{3}+\frac{2}{3}\right)l\right)b^2+\left(2a\alpha l^2t+\left(\frac{-1}{9}+\frac{4ta(\alpha+1)}{3}\right)l+\frac{2at}{3}-\frac{2}{9}\right)b+a^2t^2\left(\frac{1}{3}+\alpha l^2+\left(\frac{2\alpha}{3}+\frac{2}{3}\right)l\right)\right)^2 \quad (38)$$

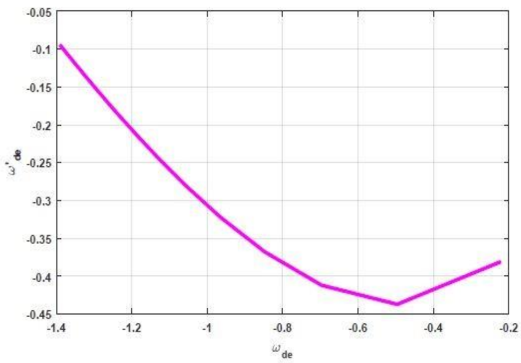


**Figure 6**: Plot of $\omega_{de}$ versus $w'_{de}$ .

Figure 6, indicates that the expansion of our universe is accelerating in freezing region.

## Conclusions

Atpresentcosmicstringsplaysavitalroletoanalyzedifferentdarkenergycosmologicalmodels. In this paper we considered non-static plane symmetric dark energy string cosmological model in the frame work of general theory of gravitation. The deceleration parameter ($q$) is positive at early age of universe and negative at presented late time which indicates decelerating phase to accelerating phase of the universe. From the graphical observation the present value of deceleration parameter $q \approx -0.73$, which coincides with the observed value. The EoS parameter ($\omega_{de}$) for the model crosses the phantom divide line $\omega_{de}$ = -1, thus it has quintom-like behavior and squared speed of sound $v^2$is positive for the model which reveals that the model with stable. We have observed from the plane analysis $\omega'_{de} < 0$ and $\omega_{de} < 0$ depicts that the region of the model lies in freezing region. We have ob- served that the values of state finder pair become $r$=1, $s$=0 at late time with standard ΛCDM model.

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