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Anisotropic Bianchi Type-*III* Cosmological Model with Wet Dark Fluid in *f* (*R,T)* Gravity

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Abstract

In this paper, we have investigated anisotropic Bianch type$-III$ cosmological model with wet dark fluid in the framework of $f(R,T)$ theory of gravity proposed by Harko *et al.,* [1] for an applicable option of the function $f(R,T) = f_1(R) + f_2(T)$, where $f_1(R) = R$ and $f_2(T) = 2\lambda T$ where λ is constant parameter. We presented exact solutions of the fields equations by using a special form of the average scale factor derived from the time varying deceleration parameter and assuming that the shear scalar (σ) in the model is proportional to expansion scalar (θ). We have seen that the model has no initial singularity and shows accelerated expansion of the universe for the late times. It is also assumed that the model gained here reveals that even in the presence of WDF, the universe indicates accelerated expansion of the universe in late times. Few physical and geometric features of the model became examined. Thus, it has been seen that $f(R,T)$ gravity explains the present phase of cosmic acceleration of our universe in the presence of wet dark fluid.

Key words: Bianchi type-III space time, Wet Dark Fluid, Hubble parameter, $f(R,T)$ gravity.

Introduction

Contemporary astronomical investigations exhibit that the recent history of the universe is persistent with accelerated expansion (Riess *et al.,* [2]; Perlmutter *et al.,* [3]; Bennet *et al.,* [4]). Researchers think this action is created by a type of negative pressure energy known as dark energy (DE). Thus dark energy plays major role in the universe acceleration. In order to observe the DE we have a few models. The first action is by developing the contented of the universe, by recommended a dark energy sector such as quintessence, phantom, chaplygin gas, *g*-essence etc. The second direction is to modify the gravitational sector itself including $f(R)$, $f(T)$, $f(R,T)$ and $f(G)$ theories of gravity.

The general theory of gravity is well established and succeeds in all localized experimental trials to the scale

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of the solar system. The modification in Einstein–Hilbert action on large cosmological scales may be a correct explanation of a late time cosmic acceleration of the expanding universe. Over recent years, developments over general relativity are attracting a lot of attention to understand the acceleration of late time and dark energy. There have been several modifications of GR, over the past decade to provide natural gravity alternative for DE. Among the various modifications, due to cosmologically important models, $f(R,T)$ gravity theory is treated as best suited.

It was proposed that interstellar acceleration could be accomplished by replacing the Einstein–Hilbert action of general relativity with a general function Ricci scalar, $f(R)$. Viable $f(R)$ gravity models have been proposed by Nojiri and Odintsov [5], Multamaki and Vilja [6, 7] and Shamir [8] which show the unification of early time inflation and late time acceleration. Harko *et al.,* [1] proposed another extension of standard general relativity, $f(R,T)$ theory of gravity where in the gravitational Lagrangian is an arbitrary function of the Ricci scalar R and of the trace of the stress energy tensor T. It is observed that the dependency from T may be caused by curious incomplete quantum effects of fluids. They have derived the field equations from a Einstein–Hilbert type variational principal and also obtained the covariant divergence of the stress-energy tensor. A particular decided filed equations eagerly achieved by $Lagrangian\ L_m$ and source term is acquired as a function of the matter $Lagrangian\ L_m$. The combination of matter and configuration and the covariant deviation for the stress energy tensor is anti-zero in the existing model.

An attractive debit is the study of Bianchi type models in modified gravity theories or alternative theories. The abnormalities found in the CMB and large-scale structure examinations of sparked increased interest in the universe anisotropic cosmological models. Kumar and Singh [9] and Singh and Agarwal [10] studied some Bianchi type-I and III cosmological models in scalar-tensor theory. Paul *et al.,* [11] obtained FRW models in $f(R)$ gravity while Sharif and Shamir [12] have studied the solutions of Bianchi type-I and V space-times in the framework of $f(R)$ gravity. Raoand Neelima [13,14] have obtained perfect fluid Einstein Rosen and Bianchi type-VI_0 universes in $f(R,T)$ gravity respectively. LRS Bianchi type-I and Bianchi type-II, $VIII$ and IX cosmological models in $f(R,T)$ theory of gravity obtained by Rao *et al.,* [15,16]. Recently, Rao *et al.,* [17] have obtained Bianchi type-III, V and VI_0 bulk viscous string cosmological models in $f(R,T)$ gravity.

Holman and Naidu [18] introduced a new candidate for DE, called the wet dark fluid (WDF). Here we confine ourselves to Bianchi type-III universe wet dark fluid in $f(R,T)$ gravity. In this paper, anisotropic Bianchi type- III cosmological model with wet dark fluid in $f(R,T)$ gravity are investigated. The paper is organized as follows: In section

2, formalism of wet dark fluid is explained. The field equations of $f(R,T)$ gravity introduction in metric format is illustrated in section 3. Explicit field equations in $f(R,T)$ gravity are derived using the particular form of $f(T)$ used by Harko *et al.,* [1] with the aid of Bianchi type-III metric in the presence of wet dark fluid in section 4. In section 5, solutions of the field equations are obtained. The physical properties of the model were examined in section 6. Ultimately, we explain the results of our work in section 7.

Wet dark fluid

Wet dark fluid (WDF) is another form of dark energy where a physically motivated equation of state is offered with the properties relevant for a DE problem. Here, we are motivated to use the WDF as candidate for dark energy model, which seems, from an empirical equation of state proposed by Tait [19] and Hayward [20], to treat water and aqueous solutions. The equation of state for the WDF is given by Holman and Naidu [18] as

$$p_{WDF} = \gamma(\rho_{WDF} - \rho^*), \quad (1)$$

The parameters γ and ρ^* are taken to be positive. p_{WDF} and ρ_{WDF} respectively represent the pressure and energy density of WDF. This non-homogeneous linear equation of state (EoS) provides a description of hydro-dynamically stability. One can notice here that the WDF of EoS contains two parts, one behaves as the usual barotropic cosmic fluid and the other behaves as a cosmological constant and unifies the dark energy and dark matter components. We are motivated to use the wet dark fluid (WDF) as a model for dark energy which stems from an empirical equation of state to treat water and aqueous solution. We treat (1) as a phenomenological equation (Chiba *et al.,* [21,22]) and for the parameters γ and ρ^*, we restrict ourselves to $0 \leq \gamma \leq 1$. Babichev *et al.,* [23] also proposed a dark energy model with a linear equation of state similar to (1), which is $p = \alpha(\rho\text{-}\rho_0)$, where α and ρ_0 are free parameters, to overcome the hydro dynamical instability of the dark energy with the usually used EoS $p = \omega\rho$ where ω =constant < 0. Motivated by the fact that this is a good approximation for many fluids, including water, in which the internal attraction of the molecules make negative pressures possible. To obtain the WDF energy density, we use the energy conservation equation

$$3H(p_{WDF} + \rho_{WDF}) + \dot{\rho}_{WDF} = 0 \quad (2)$$

where H is the average Hubble parameter given by $3H = \frac{\dot{V}}{V}$. From equation of state (1) and using $3H = \dot{V}/V$ in above equation (2), we have

$$\rho_{WDF} = \frac{\gamma}{1+\gamma}\rho^* + \frac{C_1}{V^{1+\gamma}}, \quad (3)$$

where $C_1(>0)$is the constant of integration and V is the volume expansion. The WDF naturally includes the following components: a piece that behaves as a cosmological constant as well as a standard fluid with an equation of state $p_{WDF} = \gamma\rho_{WDF}$. We can show that if we take $C_1(>0)$, this fluid will not violate the strong energy condition

$$p_{WDF} + \rho_{WDF} \geq 0,$$

$$\left.\begin{aligned} p_{WDF} + \rho_{WDF} &= (1+\gamma)\rho_{WDF} - \gamma\rho^{*} \\ &= (1+\gamma)\frac{C_1}{V^{1+\gamma}} \geq 0 \end{aligned}\right\} \quad (4)$$

Holman and Naidu [18] observed that their model is consistent with the most recent type Ia supernova, the wilkinson microwave anisotropy probe (WMAP) results as well as the constraints coming from measurements of the power spectrum. Hence they considered both the case where the dark fluid is smooth (i.e. only the cold dark matter component cluster gravitationally) as well as the case where the dark fluid also clusters. Singh and Chaubey [24] studied the Bianchi type-*I* universe with wet dark fluid. Adhav *et al.,* [25] studied the Einstein–Rosen and Bianchi type-*III* universe with wet dark fluid in general relativity. Jain *et al.,* [26] studied the axially symmetric cosmological model with wet dark fluid in the bimetric theory of gravitation. Recently, Samanta [27] discussed the Bianchi type-*V* universe filled wet dark fluid in the *f* (*R, T)* theory of gravity and showed that the universe approaches isotropy monotonically in the presence of wet dark fluid.

Gravitational field equation of $f(R,T)$ gravity

Modified theory of gravity provides a natural unification of early-time inflation and late-time acceleration (Bennet *et al.,* [4]; Capozziello [28]). Among the other modified theories, theory of scale-Gauss-Bonnet gravity, so called *f* (*G*) gravity (Nojiri *et al.,* [29]) and a theory of *f*(*T)* gravity (Linder [30]), where *T* is the torsion have been proposed to explain the accelerated expansion of universe. Harko *et al.,* [1] proposed a new *f*(*R,T*) modified theory of gravity, wherein the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar *R* and the trace of the stress-energy tensor *T* . In this paper we use the natural system of units with *G* = *c* = 1, so that the Einstein gravitational constant is defined as $\kappa^2 = 8\pi$. The field equations of modified theory of *f* (*R, T)* gravity are derived from the Hilbert–Einstein type variation principle. The action for the modified theory of *f* (*R,T)*gravity is

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\,d^4x + \int L_m\sqrt{-g}\,d^4x, \quad (5)$$

where *f* (*R,T*) is an arbitrary function of Ricci scalar *R* and *T* be the trace of stress-energy tensor (T_{ij}) of the matter. L_m is the matter Lagrangian density. The energy momentum tensor (T_{ij}) is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,L_m)}{\delta g^{ij}} \quad (6)$$

By assuming that L_m of matter depends only on the metric tensor components g_{ij}, and not on its derivatives, we obtain

$$T_{ij} = g_{ij}L_m - \frac{2\partial L_m}{\partial g^{ij}}, \quad (7)$$

Now by varying the action *S* of the gravitational field with respect to the metric tensor components g^{ij}, we obtain the field equations of *f* (*R, T)* gravity as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\nabla^i\nabla_j - \nabla_i\nabla_j\right)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}, \quad (8)$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g_{ij}\partial g^{\alpha\beta}} \quad (9)$$

Here ∇_i is the covariant derivative,

$f_R = \frac{\partial f(R,T)}{\partial R}$, $f_T = \frac{\partial f(R,T)}{\partial T}$ and T_{ij} is the standard matter energy momentum tensor derived from the Lagrangian L_m. It may be noted that when $f(R,T)= f(R)$, equation (8) yield the field equations of $f(R)$ gravity.

$$f_R(R,T)R + 3\nabla^i\nabla_j f_R(R,T) - 2f(R,T) = 8\pi T - (T + \Theta) f_T(R,T), \quad (10)$$

where $\Theta = \Theta_i^i$. Equation (10) gives a relation between Ricci scalar R and the trace of energy momentum tensor T. Then with the use of (9), we obtain the variation of stress-energy. Using matter Lagrangian L_m, the tensor of stress energy of the matter is given by

$$T_{ij} = (\rho_{WDF} + p_{WDF})u_i u_j - p_{WDF} g_{ij}, \quad (11)$$

where $u^i = (1, 0, 0, 0)$ is the four velocity and satisfies the condition $u_i u^i = 1$, where p_{WDF} and ρ_{WDF} are the energy density and pressure of the wet dark fluid respectively. Here the matter Lagrangian can be taken as $L_m = -p_{WDF}$ since, there is no unique definition of the matter Lagrangian. Then with the use of (9), we obtain for the variation of stress-energy of wet dark fluid as

$$\Theta_{ij} = -2T_{ij} - p_{WDF}\, g_{ij} \quad (12)$$

On the physical nature of the matter field, the field equations also depend through the tensor Θ_{ij}. Hence in the case of f (R,T) gravity depending on the nature of the matter source, we obtain many theoretical models corresponding to different matter contributions for modified $f(R,T)$ gravity theory. Among the different classes of Harko *et al.,* [1] models, we have considered the case $f(R, T) = f_1(R) + f_2(T)$, where $f_1(R)$ and $f_2(T)$ are the arbitrary functions of Ricci scalar R and the trace of stress-energy tensor T respectively. And if the matter source is a WDF then the gravitational field equation (8) of $f(R,T)$ gravity reduced to

$$f_1'(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} = 8\pi T_{ij} + f_2'(T)T_{ij} + \left[f_2'(T)p_{WDF} + \frac{1}{2}f_2(T)\right]g_{ij} \quad (13)$$

where prime denotes differentiation respect to argument. Similarly, if we assuming $f(R, T) = R + 2f(T)$ as a first choice where $f(T)$ is an arbitrary function of the trace of stress-energy tensor of matter, we get the gravitational field equations of $f(R, T)$ gravity from (8) as

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2(T_{ij} + \Theta_{ij})f'(T) + f(T)g_{ij} \quad (14)$$

where the prime denotes differentiation respect to argument. Since we consider the matter source is a wet dark fluid then the field equations (8) of $f(R,T)$gravity in view of (9) reduced to

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2p_{WDF} f'(T) + f(T)]g_{ij} \quad (15)$$

Metric and the filed equations

We consider spatially homogeneous and anisotropic Bianchi type-*III* metric given by

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2x}dy^2 - C^2dz^2$$

(16) where A, B and C are only functions of cosmic time t. Using co moving coordinates, (11) and (12), the $f(R, T)$ gravity field equations, with the particular choice of the function (Harko *et al.,* [1])

$$f(T) = \lambda T, \lambda = constant \quad (17)$$

for the metric (16), take the form

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = (8\pi + 3\lambda)p_{WDF} - \lambda\rho_{WDF}, \quad (18)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = (8\pi + 3\lambda)p_{WDF} - \lambda\rho_{WDF,} \quad (19)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = (8\pi + 3\lambda)p_{WDF} - \lambda\rho_{WDF,} \quad (20)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}\dot{B}}{CB} - \frac{1}{A^2} = -(8\pi + 3\lambda)\rho_{WDF} + \lambda p_{WDF,} \quad (21)$$

$$h\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \quad (22)$$

where an overhead dot denotes differentiation with respect to cosmic time *t*. Integrating equation (22), we obtain $A = g_1 B,$ (23)

where g_1 is an integration constant. Without any loss of generality, we take $g_1 = 1$, so that we have

$$A = B. \quad (24)$$

Using equation (24), the field equations (18)–(21) will reduce to

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = (8\pi + 3\lambda)p_{WDF} - \lambda\rho_{WDF,} \quad (25)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{1}{A^2} = (8\pi + 3\lambda)p_{WDF} - \lambda\rho_{WDF,} \quad (26)$$

$$\frac{\dot{A}^2}{A^2} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -(8\pi + 3\lambda)\rho_{WDF} + \lambda p_{WDF} \quad (27)$$

Subtracting equation (25) from (26) and rearranging, we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right) = \frac{1}{A^2} \quad (28)$$

Solution of the filed equations

We observe that the five independent differential equitations (18)–(22) reduced to the field equations (25)–(27) which are a system of three differential equations involving four unknown variables, namely *A*, *C*, ρ_{WDF} and p_{WDF}. Thus, a more connecting variable is needed H solve these equations. In order to obtain explicit solutions, we adopt an assumption that the shear scalar (σ) proportional to the scalar of expansion (θ) which leads to (Collins *et al.,* [31]).

$$A = C^n, \quad (29)$$

where $n \neq 0, 1$ is the positive constant parameter and maintains the anisotropic character of the space time. The expression for the scalar expansion, which is kinematical parameter, obtained by using equations (24) and (29)

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 2\frac{\dot{A}}{A} + \frac{\dot{C}}{C} = (2n+1)\frac{\dot{C}}{C} \quad (30)$$

An observational quantity which is the deceleration parameter (q) defined by $q = -\frac{a\ddot{a}}{\dot{a}^2},$ (31)

where the sign of *q* indicates whether the model inflates or not. For a decelerating model we have $q > 0$, whereas for an accelerating model of the universe $q < 0$. The field equations (18) - (22) are highly non-linear in nature. The other physical or mathematical condition to determine the solution is assuming the nature of deceleration parameter. Some authors have proposed the time dependent form of deceleration parameter and derive the different forms of the average scale factor of the model (Verma *et al.,* [32]; Akarsu and Dereli [33]; Singh [34]; Banerjee and Das [35]; Ellis and Madsen [36]; Singha and Debnath [37]; Berman [38]). Alternatively, some authors have chosen the average scale factor and then deduced the time dependent deceleration parameter. Equation (31) can also be written as

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \quad (32)$$

Abdussattar and Prajapati [39] proposed a solution for a time dependent form of deceleration parameter (*q*) as

$$q = -\frac{g_2}{t^2} + (g_3 - 1) \quad (33)$$

where $g_2 > 0$ and $g_3 > 1$ are parameters. For $g_3 = 1$, the universe will have an accelerated expansion through its revolution. Equation (32) will be integrated to give scale factor $a(t)$

$$a(t) = e^{z_1}\exp\left(\int \frac{dt}{\int(q+1)dt + z_2}\right), \quad (34)$$

where z_1 and z_2 are arbitrary constants of integrations. Substituting equation (33) into equation (34) and putting z_2 = 0, after integration we obtain

$$a(t) = e^{z_1}\left(t^2 + \frac{2g_2}{g_3}\right)^{\frac{1}{2g_2}} \quad (35)$$

For simplicity, we take z_1 = 0 and $g_3 = \frac{3}{2}$ in equation (35) so that

$$a(t) = \left(t^2 + \frac{2g_2}{3}\right)^{\frac{1}{3}} \quad (36)$$

The red shift z obtained as

$$z = -1 + \frac{a_0}{a} = -1 + \left(t^2 + \frac{2g_2}{3}\right)^{-\frac{1}{3}} \quad (37)$$

where a_0 is the present scale factor and assumed to be 1. The spatial volume (V) of the model (16) is given by

$$V = A^2C = a^3 = \left(t^2 + \frac{2g_2}{3}\right) \quad (38)$$

With the help of equation (29), the metric potentials A and C become

$$A = \left(t^2 + \frac{2g_2}{3}\right)^{\frac{n}{2n+1}} \quad (39)$$

$$C = \left(t^2 + \frac{2g_2}{3}\right)^{\frac{1}{2n+1}} \quad (40)$$

With the help of equation (39) and (40), the line element (16) reduces to

$$ds^2 = dt^2 - \left(t^2 + \frac{2g_2}{3}\right)^{\frac{2n}{2n+1}}(dx^2 + e^{-2x}dy^2) - \left(t^2 + \frac{2g_2}{3}\right)^{\frac{2}{2n+1}}dz^2 \quad (41)$$

Some physical and kinematical properties of the model

The metric given by equation (41) represents an anisotropic Bianchi type-*III* spacetime cosmological model filled with wet dark fluid in the f (R, T) gravity theory. We now discuss the physical and kinamatical behaviors of the Bianchi type-*III* cosmological model with this metric. The directional Hubble parameters (H_x H_y and H_z) and the average Hubble parameter (H) are given by

$$H_1 = \frac{\dot{A}}{A} = \frac{2nt}{(2n+1)\left(t^2 + \frac{2g_2}{3}\right)} = H_2 = \frac{\dot{B}}{B} \text{ and}$$

$$H_3 = \frac{\dot{C}}{C} = \frac{2t}{(2n+1)\left(t^2 + \frac{2g_2}{3}\right)} \quad (42)$$

$$H = \frac{1}{3}(2H_1 + H_3) = \frac{2t}{3\left(t^2 + \frac{2g_2}{3}\right)} \quad (43)$$

The expressions for the scalar expansion (ϑ), the shear scalar (σ) and the anisotropy parameter (A_m) for the metric (41) are respectively as follows: $\theta = 2\frac{\dot{A}}{A} + \frac{\dot{C}}{C} = \frac{2t}{\left(t^2 + \frac{2g_2}{3}\right)}$, (44)

$$\sigma^2 = \frac{1}{2}\left[2\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2 = \frac{4(n-1)^2t^2}{3(2n+1)^2\left(t^2 + \frac{2g_2}{3}\right)^2}$$

$$A_m = \frac{2}{3}\frac{\sigma^2}{H^2} = \frac{2(n-1)^2}{(2n+1)^2} \quad (46)$$

Subtracting equation (27) from (26) and using equation (25), the energy density (ρ_{WDF}) of wet dark fluid is

$$\rho_{WDF} = \frac{(4\pi + 2\lambda)\frac{\ddot{A}}{A} - (12\pi + 4\lambda)\frac{\dot{A}\dot{C}}{AC} - (4\pi + \lambda)\frac{\ddot{C}}{C}}{(4\pi + \lambda)(8\pi + 4\lambda)} = \frac{1}{(4\pi + \lambda)(8\pi + 4\lambda)} \times \left[\frac{2[4\pi(n-1) + \lambda(2n-1)]}{(2n+1)\left(t^2 + \frac{2g_2}{3}\right)} - \frac{8nt^2(2\pi + \lambda)(n+2)}{(2n+1)^2\left(t^2 + \frac{2g_2}{3}\right)^2}\right] \quad (47)$$

In a similar manner, we obtain the pressure p_{WDF} of wet dark fluid becomes

$$p_{WDF} = \frac{(4\pi + 2\lambda)\frac{\ddot{A}}{A} + (4\pi)\frac{\dot{A}\dot{C}}{AC} + (4\pi + \lambda)\frac{\ddot{C}}{C}}{(4\pi + \lambda)(8\pi + 4\lambda)} = \frac{1}{(4\pi + \lambda)(8\pi + 4\lambda)} \times \left[\frac{2[4\pi(n+1) + \lambda(2n+1)]}{(2n+1)\left(t^2 + \frac{2g_2}{3}\right)} - \frac{8nt^2(2\pi + \lambda)(n+2)}{(2n+1)^2\left(t^2 + \frac{2g_2}{3}\right)^2}\right] \quad (48)$$

The metric (41) together with equations (39), (40), (47) and (48) constitutes anisotropic Bianchi type-*III* cosmological model with wet dark fluid in *f* (*R, T)* gravity. It may be observed that the model (41) is free from initial singularity, i.e. at $t = 0$.

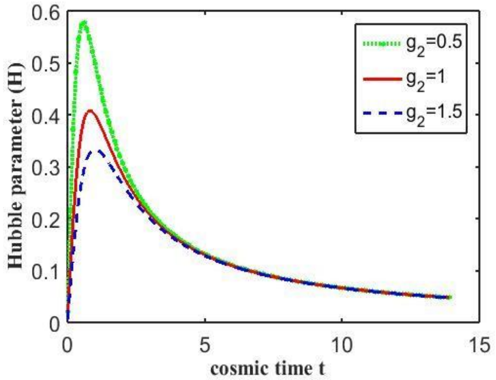

Figure 1: **P**lot of volume expansion (*V)*

Figure 1 presents the volume expansion (*V)* versus redshift (*z*) for g_2 = 0.5, 1 and 1.5. It depicts that the volume expansion increases for small values of the redshift which shows the spatial volume in this model increases as cosmic time *t* increases, which indicates the accelerated expansion of the universe. The Hubble parameter *H* starts with extremely large values and continue to decrease with passage of time which mimic the present scenario of universe (figure 2).

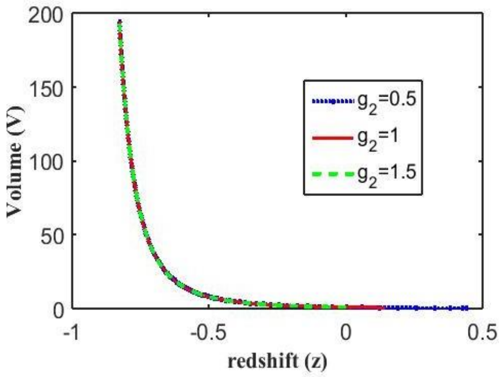

Figure 2: Plot Hubble parameter (*H*) versus redshift (*z*) for g_2 = 0.5, 1 and 1.5. versus cosmic time t for g_2 = 0.5, 1 and 1.5.

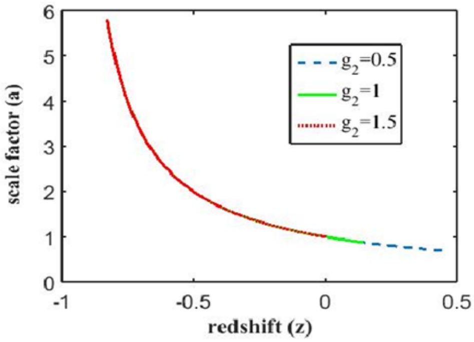

Figure 3: Plot of scale factor (*a*) versus

Figure 3 depicts the relationship between the scale factor (*a*) versus redshift (*z*). It shows that scale factor become increasing in late times. From figure 5, it is observed that the energy density of wet dark fluid (ρ_{WDF}) have positive small values in late times for chosen parameters $g_2 = 1$, $n = 2$, and $\lambda = -7$ and it vanishes for large values of cosmic time *t*.

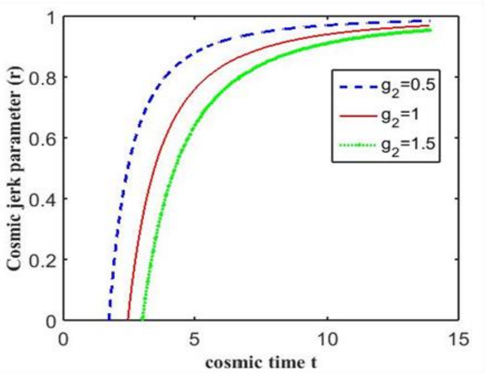

Figure 4: Plot of cosmic jerk parameter (j) redshift (*z*) for g_2 = 0.5, 1 and 1.5. versus cosmic time *t* for g_2 = 0.5, 1, 1.5 and 1.5.

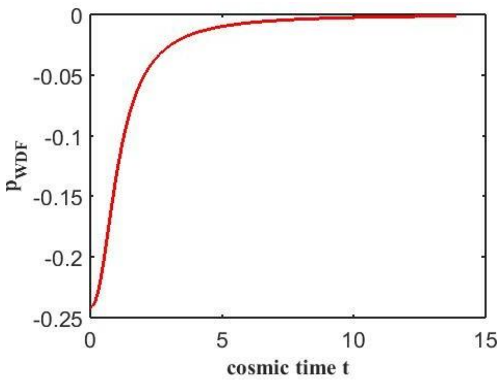

Figure 5: Plot of energy density of

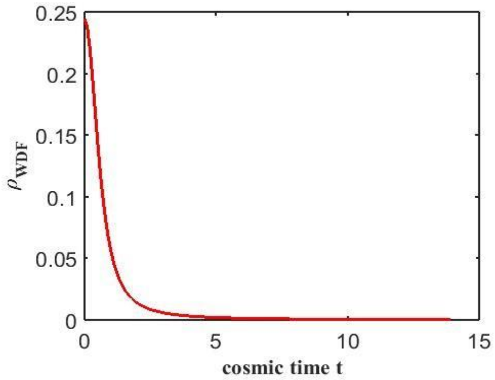

Figure 6: Plot of pressure of wet dark wet dark fluid ($\rho_{W\,DF}$) versus cosmic fluid ($p_{W\,DF}$)versus cosmic time *t* for time *t* for $g_2 = 1$, $n = 2$ and $\lambda = -7$. $g_2 = 1$, $n = 2$ and $\lambda = -7$.

Figure 6 shows the behavior of pressure of wet dark fluid ($p_{W\,DF}$) versus cosmic time *t* for $g_2 = 1$, $n = 2$, and $\lambda = -7$. The pressure of the wet dark fluid has negative values for Bianchi type- *III* space time which shows that the universe is accelerated expanding for late times with the dominance of dark energy. It is also observed that all other physical and geometrical quantities are functions of cosmic time *t* and vanish as cosmic time *t* becomes infinitely large. We see that $\frac{\sigma^2}{\theta^2} \neq 0$, and hence the model does not approach isotropy for large values of cosmic time *t*. However, the model becomes isotropic for $n = 1$ and the universe will be in a state of accelerated expansion.

Cosmic jerk parameter

It is believed that the transition from the decelerating to the accelerating phase of the universe is due to a cosmic jerk. This transition of the universe occurs for different models with a positive value of the jerk parameter and the negative value of the deceleration parameter (Visser [40]). Rapetti *et al.,* [41] showed that for flat ΛCDM model the value of jerk becomes $r = 1$. The cosmic jerk parameter is a dimensionless quantity containing the third order derivative of the average scale factor with respect to the cosmic time and it is given by

$$r = \frac{\dddot{a}}{aH^3} = q(1 + 2q) - \frac{\dot{q}}{H} \qquad (49)$$

Using equations (33) and (43) in (49), we get expression for the jerk parameter as

$$r = -\frac{4g_2g_3}{t^2} + 2{g_3}^2 - 3g_3 + 1 \qquad (50)$$

where $g_3 = \frac{3}{2}$. The jerk parameter for the model (50) has value $r = 1$ for large value of cosmic time *t*. Figure 4 shows that the cosmic jerk parameter is non-negative throughout the entire life of the universe and tends to 1 at late times for the chosen parameters. We emphasize here that the behavior of jerk parameter in the model indicates the ΛCDM limit which consistent with recent observational data of cosmology.

Conclusions

We have investigated anisotropic Bianch type-*III* cosmological model with wet dark fluid which is a candidate for dark energy model in the framework of *f*

(R, T) theory of gravity proposed by Harko *et al.,* [1] for an appropriate choice of the function $f(R, T) = f_1(R) + f_2(T)$, where $f_1(R) = R$ and $f_2(T) = 2\lambda T$ with constant parameter λ. We presented exact solutions of the fields equations by using a special form of the average scale factor derived from the time varying deceleration parameter proposed by Abdussattar and Prajapati [39] and assuming that the shear scalar (σ) in the model is proportional to expansion scalar (ϑ).

It is concluded that the model has no initial singularity. The spatial volume will be constant for cosmic time $t = 0$ which implies that the universe is expanding constantly and expands continuously approaching to infinite volume at late times. The relationship between volume of expansion versus redshift shown in figure 1 for certain parameters supports the expanding of the universe in late times. The scale factor versus redshift (figure 3) similarly supports the expansion of the universe which is consistency with recent observations of cosmological data. The behavior of the Hubble's parameter (H) versus cosmic time t has been graphed in figure 2. The parameters H, ϑ and σ^2 start with constant at initial epoch and continue to decrease with passage of time which mimic the present scenario of universe. Since $\frac{\sigma}{\theta}$=constant, the anisotropy in the universe is maintained throughout the passage of time. The energy density of wet dark fluid (ρ_{WDF}) is decreasing with increasing of cosmic time t and the pressure of the wet dark fluid (p_{WDF}) increasing negatively with cosmic time which indicates the accelerated expansion of the universe (figures 5 and 6). The pressure has negative values for Bianchi type- *III* which shows that the universe is accelerated expanding for late times. We observe that the behavior of jerk parameter in the model shows flatness of the universe as r c 1 (figure 4) for certain chosen parameters. It was proved that the modified theory of $f(R, T)$ gravity allowed transition of matter from dominated phase to an acceleration phase. Thus, it is verified that $f(R, T)$ gravity may explain the present phase of cosmic acceleration of our universe in the presence of wet dark fluid.

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